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CHANNELS THAT COOPERATIVELY SERVICE A DATA STREAM AND VOICE MES--ETC(U)

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CHANNELS THAT COOPERATIVELY SERVICE
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AND VOICE MESSAGES, I

by

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and
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A system of channels mutually accommodate both data and voice messages, voice having pre-emptive priority but being a loss system, and data being allowed to queue. Approximations to the data queue properties are derived.

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CHANNELS THAT COOPERATIVELY SERVICE A DATA STREAM AND VOICE MESSAGES*

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I. INTRODUCTION

A system of channels cooperatively services both voice and data messages arriving at one node of a communications network. This paper is devoted to the analysis of a particular channel-sharing strategy, in which voice traffic always occupies its channels when available, but data service is allowed to occur on empty voice channels. Voice traffic is taken to be of high priority; voice arrivals that find all voice channels busy are treated as losses. Note that voice traffic will be relatively infrequent as compared to data, and will also exhibit relatively long holding (service) times. Data traffic is taken to be heavy, and exhibits very short holding times (per word unit): compared to voice, data appears to arrive nearly continuously; when all data (and empty voice) channels are filled, queueing occurs.

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We present an analysis of the performance of a special type of integrated circuit and packet-switched multiplexor structure. This structure essentially occurs within the SENET network; descriptions are given by Coviello and Vena (1975), and Barbacci and Oakley (1976). In this network a time-slotted frame is utilized; a certain portion of each frame is allocated to voice traffic, while any remaining data traffic can use all remaining capacity, including that left unused by voice. Voice, on the other hand, cannot use capacity unused by data, and operates on a loss system. The subject of our analysis has the same qualitative flavor. Typical performance measures that may be calculated are (i) the loss rate of voice traffic, and (ii) the expected waiting time, or, equvialently, mean queue length, of the data.

The analysis begins with standard probabilitistic assumptions. Specifically, voice traffic arrives according to a Poisson(λ) process, and each customer has an independent exponential(μ) service time. Data arrivals are according to an independent Poisson(δ) process, and exhibit exponential (η) service times. A total of c channels are reserved for exclusive used of data, while v channels can be used by both data and voice; however, voice pre-empts data. Voice operates as an M/M/v loss system, and the well-known "Erlang B" loss formula will give the loss rate. We are mainly interested in the behavior of the data queue; however, we wish to

develop expressions for mean queue lengths for certain extreme (and realistic) parameter values. First, we will require that $\delta/\eta = \rho_d > c$. This assumption indicates that the data must be able to use excess voice capacity in order to remain stable. Second, we require that η/μ be large, perhaps on the order of 10^4. This indicates that the voice requires long service periods while the data service periods are very short.

This problem has been studied in a number of papers including Halfin and Segal (1972), Halfin (1972), Fischer and Harris (1976), Bhat and Fischer (1976), Fischer (1977), and Chang (1977). Many of these studies begin with the Kolmogorov forward equations appropriate for this system and introduce some approximations leading to a solution. While this approach is entirely appropriate, the approximations heretofore introduced have not been tailored to the $\rho_d > c$ and $\eta/\mu \approx 10^4$ situation. In fact, several of the approximations give quite misleading results in this case. We develop an approximation which is tailored to these rather extreme but realistic parameter conditions. While the Kolmogorov equations can be easily written for the Markov chain $\{(Q(t), N(t)), t > 0\}$ where $Q(t)$ represents the number of data messages in the system and $N(t)$ represents the number of voice messages at time t, the fact that both $Q(t)$ and $N(t)$ are subject to random fluctuation seems to make any direct approach to solving the equations difficult. We thus propose to treat the data as a deterministic process behaving like a fluid flow.

2. THE APPROXIMATION

To better understand the behavior of the data queue process, we first consider the special case $c = 0$, $v = 1$. In this case, the data can use the single channel only when voice traffic is not present. Consider a set of parameter values given by $\lambda = .01$, $\mu = .01$, $\delta = 25$, and $\eta = 100$. It follows that $\rho_d = .25$ and $\rho_v = 1$. The overall traffic intensity parameter $\rho = \rho_d + \rho_v(1-q)$ where q, the voice blocking probability, is .75, so the system is stable. Nevertheless, very long data queues will occasionally be created for the following simple reason. When no voice traffic is present and no data queue is present the system appears to data traffic to be an M/M/1 system with $\rho = .25$. There will be essentially no queueing at all, and this situation will persist for an average of $1/\lambda = 100$ time units. However, when a voice message arrives, the channel becomes unavailable to the data, and all data messages must now be queued. This queue will grow at a rate of 25 per unit time. Furthermore, the voice message exhibits a long holding time (on the average 100 time units), so the data queue will reach a height of 2500 on the average before it can begin to be serviced. The channel is now free, and will remain so for about 100 time units--but now the queue has 2500 customers, not zero as before. It is clear that the steady-state mean queue length is very large (2500 in fact); however, it is also clear that this classical performance evaluation measure can be very

misleading. The actual behavior of the data queue is one of long periods of essential emptiness followed by long periods of great queue length. The mean gives an average of these two extreme situations and therefore is misleading. We propose to develop approximations for this mean but to also provide other descriptions of system behavior such as idle and busy period lengths, first-passage times and steady-date distributions.

The mean queue length has been calculated exactly for $c = 0$, $v = 1$ by Fischer (1977) and is given by

$$\frac{\rho_d}{(1+\rho_v)^2 \ (1-\rho)} \left\{\frac{\eta}{\mu} \ \rho_v + (1+\rho_v)^2\right\} \tag{2.1}$$

where $\rho = \rho_d + \rho_v/(1+\rho_v)$. It is clear that if $\eta/\mu \approx 10^4$ and $\rho_v \approx 1$ as in the above example, then of the two terms in brackets η/μ will be large compared with $(1+\rho_v)^2$, hence. we can ignore this term. Ignoring this term is equivalent to ignoring the queueing that occurs when the system is empty. The analysis presented in this paper ignores terms of this type.

The fluid flow approximation is based on treating the data as a deterministic stream. Let us suppose that there are i voice channels occupied. This leaves $c + v - i$ available for data. Data arrives at rate δ and is serviced at rate $(c + v - i)\eta$ giving an overall change in the queue length of $\delta - \eta(c + v - i) = r_i$ per unit time, where $i = 0,1,\ldots,v$. It is clear that $r_0 < r_1 < \cdots < r_v$. We assume $r_0 < 0$ and

$r_v > 0$. The first is necessary for system stability while the latter follows from $\rho_d > c$. Thus there is a state N for which $r_N \leq 0 < r_{N+1}$. We treat the case $r_N < 0$, while $r_N = 0$ is a straightforward generalization. We refer to the states $0, 1, \ldots, N$ as "down" states, while $N+1, \ldots, v$ are "up" states ($0 \leq N \leq v$, so the two sets of states are nonempty). These names reflect the fact that if i voice channels are occupied, then the data queue tends to increase if i is an up state, and to decrease if i is a down state. The steady-state distribution of the occupancy of the voice channels is given by a truncated Poisson(ρ_v) distribution,

$$p_i = \frac{\rho_v^i/i!}{\sum_{j=0}^{v} \rho_v^i/i!}, \qquad 0 \leq i \leq v \tag{2.2}$$

and the loss probability $q = p_v$.

For the data queue to remain stable

$$\sum_{i=0}^{v} r_i \rho_v^i/i! < 0 .$$

If one defines $\rho = [\rho_d + \rho_v(1-q)]/(v + c)$ then the stability condition becomes $\rho < 1$.

We wish to compute a variety of quantities for the data queueing system. These quantities include

$p_{ij}(x)$ = P(voice is in state j when queue empties | voice is in state i and data in state x), $0 \leq i \leq N$, $0 \leq i \leq v$.

$\tau_i(x)$ = expected first-passage time for data from state x to state 0 starting in voice state i, $0 \leq i \leq v$.

$a_i(x)$ = expected area under data queue-length process accumulated during the first passage time to 0, $0 \leq i \leq v$.

The above quantities give important characterizations of the system performance. The first-passage times indicate the time needed to work off a backlog of size x. The area gives essentially the waiting time. If the queue is empty and the voice is in a down state then for the fluid model the queue will remain empty until the voice reaches the first up state, N+1. The queue immediately begins to grow at rate r_{N+1}. It follows that $\tau_{N+1}(0)$ represents the expected duration of the busy period. Similarly $a_{N+1}(0)$ gives the expected area accumulated during the busy period. Using renewal-theoretic ideas $a_{N+1}(0)/\tau_{N+1}(0)$ gives the mean queue length during the busy period. Similarly $p_{N+1,i}(0)$ gives the probability that the busy period will end in voice state i. The time for the voice to reach N+1 from i and hence the expected time to initiate a new busy period is easily calculated from the birth-death process. Let us designate this mean by S_i. Then $\sum_{i=0}^{N} S_i p_{N+1,i}(0) = T$ gives the expected idle time (we ignore all queueing during this period). Clearly $T/(T + \tau_{N+1}(0))$ gives the steady state data component idleness probability and

$a_{N+1}(0) \cdot T/(T + \tau_{N+1}(0))$. $\tau_{N+1}(0)$ gives the steady-state mean data queue length. It is clear that the quantities $a_i(x)$, $\tau_i(x)$, and $p_{ij}(x)$ give valuable insight into the behavior of the queueing process, incidentally providing all of the standard queueing performance measures. The special case of one down state $(N = 0)$ is easiest to handle, since in this case the p_{ij}'s can be ignored.

3. DERIVATION OF $p_{ij}(x)$ FUNCTIONS

We use a backward equation approach. Let us assume that at time $t = 0$ the queue length is $x > 0$ and i voice channels are occupied. It follows that at time dt, the new queue length will be $x + r_i\,dt$. The system will remain in state i with probability $1 - (\lambda \min(1,v-i) + i\mu)dt + o(dt)$, will move to state i+1 with probability $\lambda \min(1,v-i)dt + o(dt)$, or will move to i-1 with probability $i\mu dt + o(dt)$. Thus

$$
\begin{aligned}
p_{ij}(x) = {} & p_{ij}(x + r_i dt)(1 - (\lambda \min(1,v-i) + i\mu)dt + o(dt)) \\
& + p_{i-1\ j}(x + r_i dt)\ i\mu\ dt \\
& + p_{i+1\ j}(x + r_i dt)\lambda \min(1,v-i)dt + o(dt) \qquad (3.1)
\end{aligned}
$$

One expands the $p_{ij}(x + r_i\ dt)$ into $p_{ij}(x) + r_i p'_{ij}(x)dt + o(dt)$, collects terms and lets $dt \to 0$ to derive

$$
\begin{aligned}
& p'_{ij}(x)(-r_i) \\
& \quad = -p_{ij}(x)(\lambda \min(v-i,1) + i\mu) + p_{i-1,j}(x)i\mu \\
& \qquad + p_{i+1,j}(x)\lambda \min(v-i,1) \quad \text{with} \quad 0 \le j \le N. \qquad (3.2)
\end{aligned}
$$

If $r_N = 0$, then (3.2) indicates a linear relationship among $p_{Nj}(x)$, $p_{N-1,j}(x)$ and $p_{N+1,j}(x)$. This relationship serves to allow elimination of $p_{Nj}(x)$ and therefore allows us to assume $r_N < 0$. Equation (3.2) can be divided by $-r_i$ and the entire system rewritten in matrix form to yield

$$\underset{\sim}{P}'(x) = \frac{\mu}{\eta}\, \underset{\sim}{Q}^{*}\, \underset{\sim}{P}(x)$$

here $\underset{\sim}{P}(x) = (p_{ij}(x))$, $0 \leq i \leq v$, $0 \leq j \leq N$, a $(v+1)\times(N+1)$ stochastic matrix for each x, and $\underset{\sim}{Q}^{*}$ is defined by

$$\underset{\sim}{Q}^{*} = \begin{pmatrix} -\frac{\rho_v}{\kappa} & \frac{\rho_v}{\kappa} & & & & & \\ \frac{1}{\kappa-1} & -\frac{(1+\rho_v)}{\kappa-1} & \frac{\rho_v}{\kappa-1} & & & & \\ 0 & \frac{2}{\kappa-2} & -\frac{(2+\rho_v)}{\kappa-2} & \frac{\rho_v}{\kappa-2} & & & \\ & & & \frac{v-1}{\kappa-(v-1)} & -\frac{(v-1+\rho_v)}{\kappa-(v-1)} & \frac{\rho_v}{\kappa-(v-1)} \\ & & & & \frac{\rho_v}{\kappa-v} & -\frac{\rho_v}{\kappa-v} \end{pmatrix} \tag{3.3}$$

where $\kappa = c + v - \rho_d$. We assume $0 < \kappa < v$.

Equation (3.3) can be routinely solved to give

$$\underset{\sim}{P}(x) = \exp\left(\frac{\mu}{\eta}\, \underset{\sim}{Q}^{*}\right) \underset{\sim}{P}(0) \tag{3.4}$$

where $\exp(M) = I + M + M^2/2! + \cdots$ for a square matrix M. Interestingly, one still needs to determine $\underset{\sim}{P}(0)$ before $\underset{\sim}{P}(x)$ is fully determined.

To determine $\underset{\sim}{P}(0)$, we partition into down and up states. Thus

$$\underset{\sim}{P}(x) = \begin{pmatrix} \underset{\sim}{P}_D(x) \\ \underset{\sim}{P}_U(x) \end{pmatrix}$$

where

3. DERIVATION OF $p_{ij}(x)$ FUNCTIONS

We use a backward equation approach. Let us assume that at time $t = 0$ the queue length is $x > 0$ and i voice channels are occupied. It follows that at time dt, the new queue length will be $x + r_i\,dt$. The system will remain in state i with probability $1 - (\lambda \min(1,v-i) + i\mu)dt + o(dt)$, will move to state i+1 with probability $\lambda \min(1,v-i)dt + o(dt)$, or will move to i-1 with probability $i\mu dt + o(dt)$. Thus

$$\begin{aligned} p_{ij}(x) = {} & p_{ij}(x + r_i dt)(1 - (\lambda \min(1,v-i) + i\mu)dt + o(dt)) \\ & + p_{i-1\ j}(x + r_i dt)\, i\mu\, dt \\ & + p_{i+1\ j}(x + r_i dt)\lambda \min(1,v-i)dt + o(dt) \end{aligned} \tag{3.1}$$

One expands the $p_{ij}(x + r_i\,dt)$ into $p_{ij}(x) + r_i p'_{ij}(x)dt + o(dt)$, collects terms and lets $dt \to 0$ to derive

$$\begin{aligned} & p'_{ij}(x)(-r_i) \\ & \quad = -p_{ij}(x)(\lambda \min(v-i,1) + i\mu) + p_{i-1,j}(x)i\mu \\ & \qquad + p_{i+1,j}(x)\lambda \min(v-i,1) \quad \text{with} \quad 0 \le j \le N. \end{aligned} \tag{3.2}$$

If $r_N = 0$, then (3.2) indicates a linear relationship among $p_{Nj}(x)$, $p_{N-1,j}(x)$ and $p_{N+1,j}(x)$. This relationship serves to allow elimination of $p_{Nj}(x)$ and therefore allows us to assume $r_N < 0$. Equation (3.2) can be divided by $-r_i$ and the entire system rewritten in matrix form to yield

$$\underset{\sim}{P}_D(x) = (p_{ij}(x)), \qquad 0 \leq i \leq N, \quad 0 \leq j \leq N$$

$$\underset{\sim}{P}_U(x) = (p_{ij}(x)), \qquad N+1 \leq i \leq v, \quad 0 \leq j \leq N .$$

If no data queue is present $(x = 0)$ and i is a down state, then

$$p_{ij}(0) = \begin{cases} +1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

since emptiness is instantaneously achieved. Thus $\underset{\sim}{P}_D(0) = I$, the $(N+1) \times (N+1)$ identity matrix. It remains to calculate $\underset{\sim}{P}_U(0)$.

A second system of equations can be developed as follows. Beginning in state i (i up) and $x > 0$, one must first return to level x and then hit 0. The return to level x must occur in a down state. This allows one to write a system of "Chapman-Kolmogorov like" equations

$$\underset{\sim}{P}_U(x) = \underset{\sim}{P}_U(0)\ \underset{\sim}{P}_D(x) \qquad (3.5)$$

Equations (3.3) and (3.5) can be combined to give an expression for $\underset{\sim}{P}_U(0)$. This expression is in the form of a matrix quadratic equation:

$$(\underset{\sim}{P}_U(0),\ -\underset{\sim}{I}_{v-N})\ \underset{\sim}{Q}^{*} \begin{pmatrix} \underset{\sim}{I}_{N+1} \\ \underset{\sim}{P}_U(0) \end{pmatrix} = \underset{\sim}{0} \qquad (3.6)$$

This equation can be rewritten to be

$$\underset{\sim}{P}_U(0)\underset{\sim}{A}_{11} - \underset{\sim}{A}_{21} + \underset{\sim}{P}_U(0)\underset{\sim}{A}_{12}\underset{\sim}{P}_U(0) - \underset{\sim}{A}_{22}\underset{\sim}{P}_U(0) = \underset{\sim}{0} \tag{3.7}$$

where

$$\underset{\sim}{Q}^* = \begin{pmatrix} \underset{\sim}{A}_{11} & \underset{\sim}{A}_{12} \\ \underset{\sim}{A}_{21} & \underset{\sim}{A}_{22} \end{pmatrix} \qquad \begin{array}{l} \text{with } \underset{\sim}{A}_{11} \text{ an } (N+1)\times(N+1) \text{ matrix} \\ \text{and } \underset{\sim}{A}_{22} \text{ a } (v-N)\times(v-N) \text{ matrix} \end{array}$$

Equation (3.7) does not yield a closed form solution except in very special cases. It can, however, be solved numerically using a Newton-type iteration. Such solutions have been carried out, but the results will not be provided here.

DERIVATION OF $\tau_i(x)$ FUNCTION

The first-passage time functions can also be derived using a backward equation approach. A straightforward derivation gives

$$\begin{pmatrix} -r_0\tau_0'(x) \\ \vdots \\ -r_v\tau_v'(x) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} + \underset{\sim}{Q}\begin{pmatrix} \tau_0(x) \\ \vdots \\ \tau_v(x) \end{pmatrix} \tag{3.8}$$

or

$$\begin{pmatrix} \tau_0'(x) \\ \vdots \\ \tau_v'(x) \end{pmatrix} = \frac{1}{\eta}\begin{pmatrix} \frac{1}{\kappa} \\ \frac{1}{\kappa-1} \\ \vdots \\ \frac{1}{\kappa-v} \end{pmatrix} + \frac{\mu}{\eta}\, Q^*\begin{pmatrix} \tau_0(x) \\ \tau_1(x) \\ \vdots \\ \tau_v(x) \end{pmatrix} \tag{3.9}$$

Equation (3.9) can be solved and has an exponential solution similar to (3.4); however, the initial conditions must be determined. Letting

$$\underset{\sim}{\tau}_D(x) = \begin{pmatrix} \tau_0(x) \\ \vdots \\ \tau_N(x) \end{pmatrix} \quad \text{and} \quad \underset{\sim}{\tau}_U(x) = \begin{pmatrix} \tau_{N+1}(x) \\ \vdots \\ \tau_v(x) \end{pmatrix},$$

one can develop a Chapman-Kolmogorov relationship as follows. Beginning in an up state at level x the process must first return to x and then hit 0. It follows that

$$\underset{\sim}{\tau}_U(x) = \underset{\sim}{\tau}_U(0) + \underset{\sim}{P}_U(0)\ \underset{\sim}{\tau}_D(x) \qquad (3.10)$$

Clearly $\underset{\sim}{\tau}_D(0) = \underset{\sim}{0}$ and it remains to calculate $\underset{\sim}{\tau}_U(0)$.

Straightforward manipulations of equations (3.9) and (3.10) give

$$(\underset{\sim}{P}_U(0), -\underset{\sim}{I}_{v-N}) \begin{pmatrix} \frac{1}{\kappa} \\ \vdots \\ \frac{1}{\kappa-v} \end{pmatrix} = \mu(\underset{\sim}{P}_U(0), -\underset{\sim}{I}_{v-N})\ \underset{\sim}{Q}^* \begin{pmatrix} \underset{\sim}{0} \\ \underset{\sim}{\tau}_U(0) \end{pmatrix} \qquad (3.11)$$

where $\underset{\sim}{P}_U(0)$ has been previously determined. Using the partitioned version of

$$\underset{\sim}{Q}^* = \begin{pmatrix} \underset{\sim}{A}_{11} & \underset{\sim}{A}_{12} \\ \underset{\sim}{A}_{21} & \underset{\sim}{A}_{22} \end{pmatrix}$$

Equation (11) becomes

$$\underset{\sim}{\tau}_U(0) = \frac{1}{\mu} (\underset{\sim}{p}_U(0)\underset{\sim}{A}_{12} - \underset{\sim}{A}_{22})^{-1} (\underset{\sim}{p}_U(0), -\underset{\sim}{I}_{v-N}) \begin{pmatrix} \frac{1}{\kappa} \\ \vdots \\ \frac{1}{\kappa-v} \end{pmatrix} \tag{3.12}$$

For example, in the special case $v = 1$, $\underset{\sim}{p}_U(0) = (1)$, $\kappa = 1-\rho_d$ and

$$\underset{\sim}{\tau}_U(0) = \tau_1(0) = \frac{1}{\mu(1 + \rho_v)(1 - \rho)} . \tag{3.13}$$

In general if $N = v-1$ (only 1 up state)

$$\tau_v(0) = \frac{1}{\mu} \frac{\sum_{i=0}^{v-1} p_{vi}(0) \frac{1}{\kappa-i} - \frac{1}{\kappa-v}}{\frac{\rho_v}{\kappa-(v-1)} p_{v,v-1}(0) + \frac{v}{\kappa-v}} , \tag{3.14}$$

while if $N = 0$ (only 1 down state)

$$\underset{\sim}{\tau}_U(0) = -\frac{1}{\kappa\mu} \left(\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} (\rho_v \; 0 \cdots 0) - \underset{\sim}{A}_{22} \right)^{-1} \begin{pmatrix} \frac{1}{\kappa-1} \\ \frac{2}{\kappa-2} \\ \vdots \\ \frac{v}{\kappa-v} \end{pmatrix} \tag{3.15}$$

Explicit solutions for other cases can be written down but become very complicated.

4. DERIVATION OF $a_i(x)$ FUNCTIONS

The backward equation approach gives a straightforward derivation of the area accumulated under the queue length process during the first-passage time. One derives

$$\begin{pmatrix} -r_0 a_0'(x) \\ \vdots \\ -r_v a_v'(x) \end{pmatrix} = x \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} + \underset{\sim}{Q} \begin{pmatrix} a_0(x) \\ \vdots \\ a_v(x) \end{pmatrix} \tag{4.1}$$

Once again we define

$$\underset{\sim}{a}_D(x) = \begin{pmatrix} a_0(x) \\ \vdots \\ a_N(x) \end{pmatrix} \quad \text{and} \quad \underset{\sim}{a}_U(x) = \begin{pmatrix} a_{N+1}(x) \\ \vdots \\ a_v(x) \end{pmatrix}$$

Clearly $\underset{\sim}{a}_D(0) = \underset{\sim}{0}$, but $\underset{\sim}{a}_U(0)$ must be determined. Equation (4.1) can be rewritten as

$$\underset{\sim}{a}'(x) = \frac{1}{\eta} x \begin{pmatrix} \frac{1}{\kappa} \\ \vdots \\ \frac{1}{\kappa - v} \end{pmatrix} + \frac{\mu}{\eta} \underset{\sim}{Q}^* \underset{\sim}{a}(x) \tag{4.2}$$

which has a straightforward exponential solution once the initial conditions have been determined. To this end, a second set of equations can be found using the Chapman-Kolmogorov approach.

Beginning in an up state at level x the process must return to level x, then to 0. The expected area accumulated during the return to x is given by $a_i(0) + x\tau_i(0)$. It follows that

$$a_u(x) = a_u(0) + x\tau_u(0) + P_u(0)\ a(x). \tag{4.3}$$

Equations (4.2) and (4.3) can be combined to give

$$-\tau_U(0) = \frac{\mu}{\eta}\left(P_U(0), -I_{v-N}\right) Q^* \begin{pmatrix} 0 \\ \\ a_U(0) \end{pmatrix}. \tag{4.4}$$

By partitioning Q^* one finds

$$a_U(0) = -\frac{\eta}{\mu}\left(P_U(0)A_{12} - A_{22}\right)^{-1}\tau_U(0) \tag{4.5}$$

A simple example of the calculation involved in (4.5) is the $v = 1$ case. It can be shown that

$$a_1(0) = \frac{\eta}{\mu^2}\ \frac{\rho_d(1-\rho_d)}{(1+\rho_v)^2(1-\rho)^2} \quad \text{for } \rho < 1 \tag{4.6}$$

Carrying this a step further one can see that the idle period has mean length $1/\lambda = (1/\mu)(1/\rho_v)$. Thus recalling (3.13), we find the mean queue length given by $a_1(0)/(\tau_1(0) + 1/\lambda)$ or

$$E(Q) = \frac{\eta}{\mu} \frac{\rho_v \rho_d}{(1-\rho)(1+\rho_v)^3} \qquad \rho < 1 . \tag{4.7}$$

The $v = 1$ case can be carried further. Once $a_1(0)$ and $\tau_1(0)$ are known, $a_i(x)$ and $\tau_i(x)$ can be determined. Equation (3.11) becomes $\tau_1(x) = \tau_1(0) + \tau_0(x)$ indicating $\tau_1(x) - \tau_0(x) = \tau_1(0)$ given by (3.13). Equation (3.9) can be routinely solved to find

$$\begin{aligned} \tau_0(x) &= \frac{1}{\eta(1-\rho_d)} \left(1 + \frac{\rho_v}{(1+\rho_v)(1-\rho)}\right) x \\ \tau_1(x) &= \frac{1}{\mu} \left[\frac{\mu}{\eta(1-\rho_d)} \left(1 + \frac{\rho_v}{(1+\rho_v)(1-\rho)}\right) x + \left(\frac{1}{(1+\rho_v)(1-\rho)}\right) \right] \end{aligned} \tag{4.8}$$

The area function can also be explicitly determined. Equation (4.3) becomes $a_1(x) - a_0(x) = x\tau_1(0) + a_1(0)$. Substituting this into (4.1) gives

$$\begin{aligned} a_0(x) &= \frac{x^2}{2} \left(\frac{1}{\kappa} + \tau_1(0)\right) + \frac{\rho_v}{\kappa} \frac{\mu}{\eta} a_1(0) x \\ a_1(x) &= \frac{x^2}{2} \left(\frac{1}{\kappa} + \tau_1(0)\right) + \left(\frac{\rho_v}{\kappa} \frac{\mu}{\eta} a_1(0) + \tau_1(0)\right) x + a_1(0), \end{aligned} \tag{4.9}$$

all coefficients of which have been previously determined. In the special case $\rho_d = 1/4$, $\rho_v = 1$, $\rho = 3/4$ mentioned earlier with $\lambda = .01$, $\mu = .01$, $\delta = 25$, $\eta = 100$ then

$$\tau_0(x) = \frac{x}{25}$$

$$\tau_1(x) = 4x + 200$$

$$a_0(x) = \frac{602}{3} x^2 + 100x \tag{4.10}$$

$$a_1(x) = \frac{602}{3} x^2 + 300x + 750{,}000$$

$$E(Q) = \frac{a_1(0)}{\tau_1(0) + \frac{1}{\mu}\frac{1}{\rho_v}} = \frac{750{,}000}{200 + 100} = 2500 .$$

The voice loss rate is given by $\lambda\rho_v/(1+\rho_v) = \lambda/2 = .005$.

5. STEADY-STATE DISTRIBUTION OF DATA QUEUE LENGTH

One can use a forward equation approach to develop an equilibrium distribution for the data queue length. Define $p(x,j,t)$ to be the probability of j voice channels occupied and x data units in the system at time t. It is easily seen that for $x > 0$ and dt small

$$p(x,j,t + dt)$$

$$= p(x-r_j dt,j,t)(1 - (\lambda + j\mu)dt) + p(x-r_{j-1}dt,\ j-1,t)\lambda dt$$
$$+ p(x-r_{j+1}dt,j+1,t)\ (j+1)\mu\, dt + o(dt)\ , \quad 0 \leq j \leq v$$

where $p(x,-1,t) \equiv 0$.

Standard manipulations that treat x as a continuous variable lead to these equations

$$r_j \frac{\partial p}{\partial x} + \frac{\partial p}{\partial t} = -(\lambda \min(1,v-j) + j\mu)\ p(x,j,t)$$
$$+ \lambda\, p(x,j-1,t) + (j+1)\mu\ p(x,j+1,t), \quad x > 0. \tag{5.1}$$

Setting $t \to \infty$ and assuming $p(x,j,t) \to p_j(x)$ and $[\partial p(x,j,t)]/\partial t \to 0$, we find

$$r_j\ p_j'(x) = -(\lambda \min(1,v-j) + j\mu)\ p_j(x)$$
$$+ \lambda\ p_{j-1}(x) + (j+1)\mu\ p_{j+1}(x), \quad 0 \leq j \leq v \tag{5.2}$$

with $x > 0$, $p_{-1}(x) = p_{v+1}(x) = 0$. The prime denotes x-derivatives.

Equation (5.2) is incomplete as it does not contain information about the boundary behavior.

Equation (5.2) can be summarized in matrix form by

$$\underset{\sim}{P}'(x) = -\underset{\sim}{P}(x)\ R^{*}\frac{\mu}{\eta}, \qquad x > 0 \tag{5.3}$$

where $\underset{\sim}{P}(x) = (p_0(x), \ldots, p_v(x))$ and

$$R^{*} = \begin{pmatrix} \frac{-\rho_v}{\kappa} & \frac{\rho_v}{\kappa-1} & & & & 0 \\ \frac{1}{\kappa} & \frac{-(1+\rho_v)}{\kappa-1} & \frac{\rho_v}{\kappa-2} & & & \\ & \frac{2}{\kappa-1} & \frac{-(2+\rho_v)}{\kappa-2} & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & 0 & & \ddots & \frac{-(v-1+\rho_v)}{\kappa-(v-1)} & \frac{\rho_v}{\kappa-v} \\ & & & & \frac{v}{\kappa-(v-1)} & \frac{-v}{\kappa-v} \end{pmatrix}$$

Equation (3) can be routinely solved to get

$$\underset{\sim}{P}(x) = \underset{\sim}{c}\ \exp(-\frac{\mu}{\eta}R^{*}x) \tag{5.4}$$

with $\underset{\sim}{c} = (c_0, c_1, \ldots, c_v)$. The constants c must be determined, and equation (5.4) gives only the density function, not the mass at the boundary. In view of the fluid flow approximation, there will be mass at 0, given by π_i, for each down state i, $0 \leq i \leq N$, however, no mass at the boundary for any up state, $N+1 \leq i \leq v$. Furthermore, the equilibrium distribution over i is given by $(\rho_v^i/i!)/\sum_{j=0}^{v}\rho_v^j/j!$ It follows that

$$\int_0^\infty p_i(x)\,dx + \pi_i = (\rho_v^i/i!)/\sum_{n=0}^{v} \rho_v^j/i!, \qquad 0 \le i \le N$$

$$\int_0^\infty p_i(x)\,dx = (\rho_v^i/i!)/\sum_{j=0}^{v} \rho_v^j/j!, \qquad N+1 \le i \le v \tag{5.5}$$

It remains to determine $\underset{\sim}{c}$ and $(\pi_0,\ldots,\pi_N)$. Let $\underset{\sim}{R}^* = \underset{\sim}{\Phi D \psi}$ with $\underset{\sim}{\psi\Phi} = \underset{\sim}{I}$ and

$$\underset{\sim}{D} = \begin{pmatrix} 0 & & & \\ & \alpha_1 & & \\ & & \ddots & \\ & & & \alpha_v \end{pmatrix}.$$

where we order the eigenvalues such that $\alpha_{N+1},\ldots,\alpha_v > 0$ while $\alpha_1,\ldots,\alpha_N < 0$. It is clear that (5.4) can be rewritten to give

$$\underset{\sim}{p}(x) = \underset{\sim}{c}\Phi \begin{pmatrix} 1 & & & \\ & \exp(-\frac{\mu}{\eta}x\alpha_1) & & \\ & & \ddots & \\ & & & \exp(-\frac{\mu}{\eta}x\alpha_v) \end{pmatrix} \psi \tag{5.6}$$

The functions $p_i(x)$ are linear combinations of the $\exp(-\frac{\mu}{\eta}x\alpha_i)$. In order for these functions to be integrable, the coefficients associated with those α_i which are negative must be 0. This provides constraints on the $\underset{\sim}{c}$. If $\Phi = (\underset{\sim}{\phi}_0,\underset{\sim}{\phi}_1,\ldots,\underset{\sim}{\phi}_v)$ whose columns are right eigenvectors, then $\underset{\sim}{c}\underset{\sim}{\phi}_i = 0$, $0 \le i \le N$. The remaining equations governing $\underset{\sim}{c}$ come from (5.5). Letting $c\phi_i = 0$ for $0 \le i \le N$ we have

$$p_i(x) = \sum_{j=N+1}^{v} \underset{\sim}{c}\phi_j \psi_{ji} \exp(-\frac{\mu}{\eta} x\alpha_j), \quad N+1 \le i \le v,$$

and (5.7)

$$\int_0^\infty p_i(x)dx = \frac{\rho_v^i/i!}{\sum_{k=0}^{v}(\rho_v^k/k!)} = \frac{\eta}{\mu} \sum_{j=N+1}^{v} \underset{\sim}{c}\phi_j \psi_{ji}/\alpha_j \ .$$

This gives v+1 independent equations which determine $\underset{\sim}{c}$. Once $\underset{\sim}{c}$ has been determined, $p_i(x)$, $0 \le i \le N$ are determined by (5.4). One can now determine $\pi_0, \ldots, \pi_N$, the boundary probabilities, using (5.5). The equilibrium distribution is now completely determined.

Let us consider the special case $v = 1$. Equation (1) becomes

$$p_0'(x) = -\frac{\mu}{\eta}\left(-\frac{\rho_v}{\kappa} p_0(x) + \frac{1}{\kappa} p_1(x)\right)$$

$$p_1'(x) = -\frac{\mu}{\eta}\left(\frac{\rho_v}{\kappa-1} p_0(x) - \frac{1}{\kappa-1} p_1(x)\right) \qquad (5.8)$$

$$kp_0'(x) + (k-1)p_1'(x) = 0,$$

hence,

$$kp_0(x) + (k-1)p_1(x) = 0 \quad \text{and} \quad p_1(x) = -\frac{k}{k-1} p_0(x).$$

Substitution into (8) yields

$$p_0'(x) = -\frac{\mu}{\eta}\left(\frac{-\rho_v}{\kappa} - \frac{1}{\kappa-1}\right) p_0(x) = -\frac{\mu}{\eta}\frac{(1-\rho)(1+\rho_v)}{\rho_d(1-\rho_d)} p_0(x)$$

$$\therefore \quad p_0(x) = c \exp\left[\frac{\mu}{\eta}\frac{(1-\rho)(1+\rho_v)}{\rho_d(1-\rho_d)}\right] \tag{5.9}$$

$$p_1(x) = c\,\frac{1-\rho_d}{\rho_d} \exp\left[-\frac{\mu}{\eta}\frac{(1-\rho)(1+\rho_v)}{\rho_d(1-\rho_d)}\, x\right]$$

Now

$$\int_0^\infty p_1(x)\,dx = \frac{\rho_v}{1+\rho_v}$$

thus

$$\frac{\rho_v}{1+\rho_v} = c\,\frac{(1-\rho_d)}{\rho_d}\,\frac{\eta}{\mu}\,\frac{\rho_d(1-\rho_d)}{(1-\rho)(1+\rho_v)} \quad \text{and} \quad c = \frac{\rho_v(1-\rho)}{(1-\rho_d)^2}\,\frac{\mu}{\eta}\,. \tag{5.10}$$

It is then possible to determine π_0 by means of the relation

$$\pi_0 = \frac{1}{1+\rho_v} - \int_0^\infty p_0(x)\,dx \quad .$$

Finally we find the marginal queue length distribution in a simple explicit form

$$p(x) = \begin{cases} \dfrac{\mu}{\eta}\,\dfrac{\rho_v(1-\rho)}{(1-\rho_d)^2\rho_d} \exp\left[-\dfrac{\mu}{\eta}\dfrac{(1-\rho)(1+\rho_v)}{\rho_d(1-\rho_d)}\, x\right] & x > 0 \\[2em] \dfrac{1-\rho}{1-\rho_d} & x = 0 \end{cases} \tag{5.11}$$

The mean queue length is given by

$$E(Q) = \left(\frac{\eta}{\mu}\right)\frac{\rho_d\rho_v}{(1-\rho)(1+\rho_v)^2} \qquad \rho < 1. \tag{5.12}$$

6. GENERALIZATIONS

The preceding analysis giving the data queue length can be carried out in greater generality. One may wish to view voice transmissions as a stream of alternating bursts and dead times. At any time, t, a voice customer assigned to a particular channel will be in one of two states: active transmission or inactive. During an inactive period the channel could be used for data transmissions. Such a strategy could greatly increase the data capacity, or reduce the data queue length, or both. One might assume a voice user moves between the active and inactive states in a Markovian way. As a result, one can define voice states $\{1, \ldots, M\}$ and a continuous time Markov chain with generator $\underset{\sim}{Q}$ describing the movement among these states. For each voice state i the number of channels available for data transmission can be found, and a rate of increase or decrease in the data queue length determined. Let that rate be denoted by r_i, and assume that the voice states are labelled in such a way that $r_1 \leq r_2 \leq \cdots \leq r_M$. We would assume there is a state I for which $r_I < 0 < r_{I+1}$. Up and down states can now be defined. The analysis carried out in previous sections can be shown to hold for this more general situation. The expression $\mu \underset{\sim}{Q}^*$ must be replaced by $\underset{\sim}{Q}$, while the expression

$$\eta \begin{pmatrix} \frac{1}{\kappa} \\ \frac{1}{\kappa-1} \\ \vdots \\ \frac{1}{\kappa-v} \end{pmatrix}$$

must be replaced by

$$\begin{pmatrix} \frac{1}{r_1} \\ \vdots \\ \frac{1}{r_N} \end{pmatrix} .$$

It follows that first passage times, queue lengths, and busy period lengths can be determined for this more general problem.

A second generalization involves a voice limitation procedure. One way to prevent the buildup of very long data queues is to increase the number of voice channels. This can be accomplished by providing lower transmission rates on certain voice channels, for example, one might provide 10 8kBPS channels; however, when 8 of those are in use one might divide the remaining 2 into 4 2kBPS channels. This results in lower quality transmission but less delay. Again this situation can be modelled by assuming various voice states $\{1, \ldots, N\}$ and movement among them according to a Markov chain with generator Q. Each voice state determines a data queue rate r_i. Again the preceding analysis can be applied directly to this case. One can therefore study the tradeoffs between data queue length (and delays), voice blocking probabilities, and voice transmission quality from formulas in this paper.

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